3.1 Skill Builder

Divisibility Rules

We can use rules to find out if a number is a factor of another number.

To find out if 2, 4, 5, 8, or 10 is a factor, look at the last digits in the number:

- 2 is a factor of 35**4** because the last digit, 4, is even.
- 4 is a factor of 5**24** because 4 is a factor of the last two digits, 24.
- 5 is a factor of 58**5** because the last digit is 5.
- 8 is a factor of 3**400** because 8 is a factor of the last three digits, 400.
- 10 is a factor of 21**0** because the last digit is 0.

A number that ends in 5 or 0 is divisible by 5.

To find out if 3 or 9 is a factor, add the digits in the number:

- 3 is a factor of 411 because 3 is a factor of the sum of the digits: $4 + 1 + 1 = 6$
- 9 is a factor of 747 because 9 is a factor of the sum of the digits: $7 + 4 + 7 = 18$

To find out if 6 is a factor, use the rules for 2 and 3:

- 6 is a factor of 216 because the last digit, 6, is even, and 3 is a factor of the sum of the digits: $2 + 1 + 6 = 9$

Check

1. Write yes or no to answer each question.

a) Is 2 a factor of 457?

Is the last digit even? _____

So, is 2 a factor of 457? _____

b) Is 3 a factor of 732?

The sum of the digits is _____.

So, is 3 a factor of 732? _____

c) Is 5 a factor of 734?

Is the last digit 5 or 0? _____

So, is 5 a factor of 734? _____

d) Is 4 a factor of 712?

The last 2 digits are _____.

Is 4 a factor of 12? _____

Is 4 a factor of 712? _____

e) Is 6 a factor of 558?

Is the last digit even? _____

The sum of the digits is _____.

Is 3 a factor of 558? _____

So, is 6 a factor of 558? _____

f) Is 8 a factor of 1064?

The last 3 digits are _____.

Is 8 a factor of 64? _____

So, is 8 a factor of 1064? _____

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Exercises 3.1

A

3. List the first 6 multiples of each number.

a) 6 b) 13 c) 22
d) 31 e) 45 f) 27

4. List the prime factors of each number.

a) 40 b) 75 c) 81
d) 120 e) 140 f) 192

5. Write each number as a product of its prime factors.

a) 45 b) 80 c) 96
d) 122 e) 160 f) 195

B

6. Use powers to write each number as a product of its prime factors.

a) 600 b) 1150
c) 1022 d) 2250
e) 4500 f) 6125

7. Explain why the numbers 0 and 1 have no prime factors.

8. Determine the greatest common factor of each pair of numbers.

a) 46, 84 b) 64, 120
c) 81, 216 d) 180, 224
e) 160, 672 f) 220, 860

9. Determine the greatest common factor of each set of numbers

a) 150, 275, 420 b) 120, 960, 1400
c) 126, 210, 546, 714 d) 220, 308, 484, 988

10. Determine the least common multiple of each pair of numbers.

a) 12, 14 b) 21, 45
c) 45, 60 d) 38, 42
e) 32, 45 f) 28, 52

11. Determine the least common multiple of each set of numbers.

a) 20, 36, 38 b) 15, 32, 44
c) 12, 18, 25, 30 d) 15, 20, 24, 27

12. Explain the difference between determining the greatest common factor and the least common multiple of 12 and 14.

13. Two marching bands are to be arranged in rectangular arrays with the same number of columns. One band has 42 members, the other has 36 members. What is the greatest number of columns in the array?

14. When is the product of two numbers equal to their least common multiple?

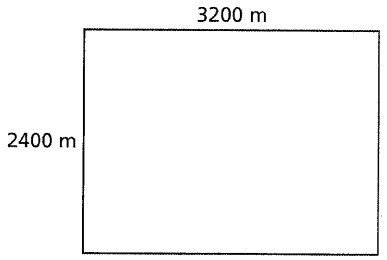
15. How could you use the greatest common factor to simplify a fraction? Use this strategy to simplify these fractions.

a) $\frac{185}{325}$ b) $\frac{340}{380}$ c) $\frac{650}{900}$
d) $\frac{840}{1220}$ e) $\frac{1225}{2750}$ f) $\frac{2145}{1105}$

16. How could you use the least common multiple to add, subtract, or divide fractions? Use this strategy to evaluate these fractions.

a) $\frac{9}{14} + \frac{11}{16}$ b) $\frac{8}{15} + \frac{11}{20}$
c) $\frac{5}{24} - \frac{1}{22}$ d) $\frac{9}{10} + \frac{5}{14} + \frac{4}{21}$
e) $\frac{9}{25} + \frac{7}{15} - \frac{5}{8}$ f) $\frac{3}{5} - \frac{5}{18} + \frac{7}{3}$
g) $\frac{3}{5} \div \frac{4}{9}$ h) $\frac{11}{6} \div \frac{2}{7}$

17. A developer wants to subdivide this rectangular plot of land into congruent square pieces. What is the side length of the largest possible square?



18. Do all whole numbers have at least one prime factor? Explain.

19. a) What are the dimensions of the smallest square that could be tiled using an 18-cm by 24-cm tile? Assume the tiles cannot be cut.

b) Could the tiles in part a be used to cover a floor with dimensions 6.48 m by 15.12 m? Explain.

20. The Dominion Land Survey is used to divide much of western Canada into sections and acres. One acre of land is a rectangle measuring 66 feet by 660 feet.

a) A section is a square with side length 1 mile. Do the rectangles for 1 acre fit exactly into a section? Justify your answer.
[1 mile = 5280 feet]

b) A quarter section is a square with side length $\frac{1}{2}$ mile. Do the rectangles for 1 acre fit exactly into a quarter section? Justify your answer.

c) What is the side length of the smallest square into which the rectangles for 1 acre will fit exactly?

C

21. Marcia says that she knows that 61 is a prime number because she tried dividing 61 by all the natural numbers up to and including 7, and none of them was a factor. Do you agree with Marcia? Explain.

22. A bar of soap has the shape of a rectangular prism that measures 10 cm by 6 cm by 3 cm. What is the edge length of the smallest cube that could be filled with these soap bars?

ANSWERS: 3.1

3.1 Factors and Multiples of Whole Numbers, page 140

3. a) 6, 12, 18, 24, 30, 36
b) 13, 26, 39, 52, 65, 78
c) 22, 44, 66, 88, 110, 132
d) 31, 62, 93, 124, 155, 186
e) 45, 90, 135, 180, 225, 270
f) 27, 54, 81, 108, 135, 162

4. a) 2, 5 b) 3, 5
c) 3 d) 2, 3, 5
e) 2, 5, 7 f) 2, 3

5. a) $3 \cdot 3 \cdot 5$, or $3^2 \cdot 5$
b) $2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$, or $2^4 \cdot 5$
c) $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$, or $2^5 \cdot 3$
d) $2 \cdot 61$
e) $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$, or $2^5 \cdot 5$
f) $3 \cdot 5 \cdot 13$

6. a) $2^3 \cdot 3 \cdot 5^2$ b) $2 \cdot 5^2 \cdot 23$
c) $2 \cdot 7 \cdot 73$ d) $2 \cdot 3^2 \cdot 5^3$
e) $2^2 \cdot 3^2 \cdot 5^3$ f) $5^3 \cdot 7^2$

8. a) 2 b) 2^3, or 8
c) 3^3, or 27 d) 2^2, or 4
e) 2^5, or 32 f) $2^2 \cdot 5$, or 20

9. a) 5 b) $2^3 \cdot 5$, or 40
c) $2 \cdot 3 \cdot 7$, or 42 d) 2^2, or 4

10. a) $2^2 \cdot 3 \cdot 7$, or 84 b) $3^2 \cdot 5 \cdot 7$, or 315
c) $2^2 \cdot 3^2 \cdot 5$, or 180 d) $2 \cdot 3 \cdot 7 \cdot 19$, or 798
e) $2^5 \cdot 3^2 \cdot 5$, or 1440 f) $2^2 \cdot 7 \cdot 13$, or 364

11. a) $2^2 \cdot 3^2 \cdot 5 \cdot 19$, or 3420
b) $2^5 \cdot 3 \cdot 5 \cdot 11$, or 5280
c) $2^2 \cdot 3^2 \cdot 5^2$, or 900
d) $2^3 \cdot 3^3 \cdot 5$, or 1080

12. Greatest common factor: 2;
least common multiple: $2^2 \cdot 3 \cdot 7$, or 84

13. $2 \cdot 3$, or 6

14. The greatest common factor of the two numbers is 1.

15. a) $\frac{37}{65}$ b) $\frac{17}{19}$
c) $\frac{13}{18}$ d) $\frac{42}{61}$
e) $\frac{49}{110}$ f) $\frac{33}{17}$

16. a) $\frac{149}{112}$ b) $\frac{65}{60}$, or $\frac{13}{12}$
c) $\frac{43}{264}$ d) $\frac{304}{210}$, or $\frac{152}{105}$
e) $\frac{121}{600}$ f) $\frac{239}{90}$
g) $\frac{27}{20}$ h) $\frac{77}{12}$

17. 800 m

18. No; 1 does not have any prime factors.

19. a) 72 cm by 72 cm b) Yes

20. a) Yes b) Yes
c) 660 feet

21. Yes

22. 30 cm

Exercises 3.2

A

4. Determine the square root of each number. Explain the process used.
a) 196 b) 256 c) 361 d) 289 e) 441

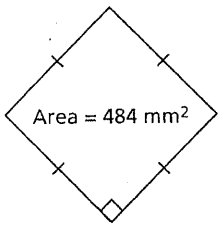
5. Determine the cube root of each number. Explain the process used.
a) 343 b) 512 c) 1000 d) 1331 e) 3375

B

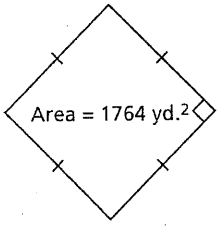
6. Use factoring to determine whether each number is a perfect square, a perfect cube, or neither.
a) 225 b) 729 c) 1944
d) 1444 e) 4096 f) 13 824

7. Determine the side length of each square.

a)

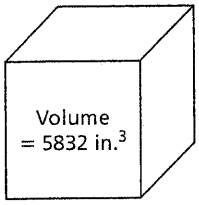


b)

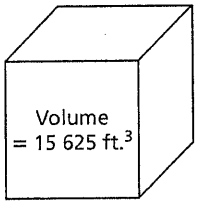


8. Determine the edge length of each cube.

a)



b)



9. In February 2003, the Battlefords Chamber of Commerce in Saskatchewan placed a cage containing a 64-cubic foot ice cube along Yellowhead Highway. Local customers were asked to predict when the ice cube would melt enough for a ball above the ice cube to fall through it. What was the surface area of the cube?

ANSWERS

3.2 Perfect Squares, Perfect Cubes, and Their Roots, page 146

4. a) 14 **b)** 16
c) 19 **d)** 17
e) 21

5. a) 7 **b)** 8
c) 10 **d)** 11
e) 15

6. a) Perfect square
b) Perfect square and perfect cube
c) Neither
d) Perfect square
e) Perfect square and perfect cube
f) Perfect cube

7. a) 22 mm **b)** 42 yd.

8. a) 18 in. **b)** 25 ft.

9. 96 ft.^{2}